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ORIGINAL PAPER

The fallow and the non-fallow states in swidden agriculture: a stochastic analysis

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Abstract At any point in time, a cleared parcel of forest land (CPFL) used for swidden agriculture exists in either the *fallow* or in the *non-fallow* state. Further, the practice of swidden agriculture requires one to operate in an environment of *uncertainty*. These two points notwithstanding, there are virtually no probabilistic models of swidden agriculture that explicitly account for the above dichotomy. Hence, in this paper, we use a stochastic model and a long run perspective to shed light on two hitherto unstudied questions concerning a CPFL used for swidden agriculture. First, we use renewal theory to determine the long run fraction of time that our CPFL is in either the fallow or in the non-fallow state. Second, we use the hyperexponential distribution to compute the stationary probability that the excess variable associated with the stochastic process representing our CPFL exceeds a given value.

Keywords Fallow state · Non-fallow state · Stochastic process · Swidden agriculture

JEL Classification Q150 · C440

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1 Introduction

Swidden agriculture—also known as slash-and-burn agriculture and as shifting cultivation—is practiced by small farmers in many tropical developing countries. As a result, there is now a sizeable empirical and case study based literature on this kind of agriculture. Belsky and Siebert (2003) have studied the ways in which the conversion of swidden fields to sun grown cacao constrains future food production opportunities in Sulawesi, Indonesia. Batabyal and Lee (2003) have computed the optimal length of time during which cleared land ought to be left fallow by swidden cultivators. DeAngelo and Batabyal (2004) have analyzed the usefulness of "fertilizer use" and "no fertilizer use" policies for overseeing the problem of soil fertility deterioration on a cleared parcel of forest land (CPFL) used for swidden agriculture. Doolittle (2004) has noted that contrary to popular belief, at the time of contact with Europeans, native food producers in the eastern woodlands of North America did not practice swidden agriculture. Thimmappa and Mahesh (2006) have shown that what they call "conservation farming" can be a useful substitute for shifting cultivation in Meghalaya, India. Tschakert et al. (2007) have argued that in Panama, there is considerable biophysical potential for carbon offsets in small scale slash-and-burn agriculture through longer fallow periods and better fallow management. Balsdon (2007) has used a dynamic model to argue that poverty reduction among shifting cultivators will lead to the accelerated extraction of a natural resource and also to a longer extraction period. Finally, Baird and Shoemaker (2007) have chronicled the ill effects arising from the resettlement of swidden cultivators from remote highlands to lowland areas in Laos.

The papers discussed in the previous paragraph have certainly advanced our understanding of the many intricacies surrounding the practice of swidden agriculture. In the context of our paper, two aspects of this advanced understanding are germane. First, it is clear that at any given point in time, a CPFL used for swidden agriculture exists in either the *fallow* state or in the *non-fallow* state. Second, it is also clear that the practice of swidden agriculture requires one to operate in an environment of *uncertainty*. These two points notwithstanding, there are virtually no probabilistic models of swidden agriculture that explicitly account for the above dichotomy. Therefore, in this paper, we use a stochastic model and a long run perspective to shed light on two hitherto unstudied questions concerning a CPFL used for swidden agriculture. However, before we proceed to these two questions, let us first comprehend the five essential stages in the swidden cycle.

Following Batabyal and Beladi (2004), these five stages are as follows. First, forest trees are cut, the debris is cleared, and the cut growth is burned. The burning of the forest vegetation clears the ground for planting and releases important nutrients. As the burned vegetation decays, the organic levels in the soil rise and this increases the soil's fertility. Second, before the rains cause soil erosion and before the ash bed can be blown or leached away, planting begins. Third, with the onset of the rainy season, normal precipitation leads to rapid plant growth. Fourth, during the harvesting season, farmers protect the crop from pests and they routinely use simple instruments such as finger knives to harvest the grain. Finally, the CPFL is left fallow. Within a couple of years, land quality improves and a closed canopy of secondary forest develops. If the CPFL is left fallow for a sufficiently long period of time then nutrients will revert back to the soil and this will permit the above delineated swidden cycle to be repeated.

This description should convince the reader that the time spent by the CPFL during the first four stages of the swidden cycle is the time spent in the non-fallow state. In addition, the time spent by the CPFL in the fifth and final stage is the time spent in the fallow state. Note that it is not necessary for the time spent by the CPFL in the fallow state to exceed the time spent in the non-fallow state. With this discussion in place, we are now in a position to clearly state the two objectives of this paper. Our first objective is to use renewal theory in general and the theory of renewal-reward processes in particular to determine the long run fraction of time that our CPFL is in either the fallow or in the non-fallow state.[1] Our second objective is to use the hyperexponential distribution function to compute the long run or stationary probability that the excess variable associated with the stochastic process representing our CPFL exceeds a given non-negative value.

The rest of this paper is organized as follows. First, Sect. 2.1 describes the renewal-reward theorem that will form the centerpiece for much of our discussion of the first objective delineated in the previous paragraph. Second, Sect. 2.2 determines the long run fraction of time that our CPFL is in either the fallow or in the non-fallow state. Third, Sect. 2.3 computes the stationary probability that the excess variable associated with the stochastic process representing our CPFL is greater than a given non-negative value. Finally, Sect. 3 concludes and discusses potential extensions of the research delineated in this paper.

2 The CPFL and its stochastic properties

2.1 Preliminaries

From Ross (2003, pp. 416–425) we know that a stochastic process $\{Z(t) : t \geq 0\}$ is a counting process if $Z(t)$ denotes the total number of counts that have taken place by time t. Clearly, since $Z(t-1)$, $Z(t)$, $Z(t+1)$, etc. are stochastic, the time between any two counts $Z(t)$ and $Z(t-1)$ is also stochastic. This time between any two counts is called the interoccurrence time. A counting process for which the interoccurrence times have a *general* cumulative probability distribution function is a renewal process.

Consider a renewal process $\{Z(t) : t \geq 0\}$ with interoccurrence times X_z, $z \geq 1$, which have a cumulative probability distribution function $G(\cdot)$. In addition, assume that a monetary reward R_z is earned when the zth renewal is completed. Let $R(t)$, the total reward earned by time t, be $\sum_{z=1}^{Z(t)} R_z$, and let $E[R_z] = E[R]$, and $E[X_z] = E[X]$. The assumptions $E[R_z] = E[R]$ and $E[X_z] = E[X]$ are standard in formal statements of the renewal-reward theorem. See, for instance, Ross (2003, p. 417). Specifically, these two assumptions tell us that in expected value terms, the rewards earned at the various renewals and the lengths of the various renewal intervals, are equal. With this background, the renewal-reward theorem—see Ross (2003, p. 417)

[1] See Batabyal and Nijkamp (2008) and Rohlin and Batabyal (2005) for examples of stochastic modeling in other economic contexts.

or Tijms (2003, p. 41)—tells us that if $E[R]$ and $E[X]$ are finite, then with probability one,

$$\lim_{t\to\infty} \frac{E[R(t)]}{t} = \frac{E[R]}{E[X]}. \tag{1}$$

In words, (1) is telling us that if we think of a cycle being completed every time a renewal occurs, then the long run expected reward—the left-hand-side (LHS) of (1)—is the expected reward in a cycle or E[reward per cycle] $= E[R]$ divided by the expected amount of time it takes to complete that cycle or E[length of cycle] $= E[X]$. The reader should understand that the renewal-reward theorem describes a *long run* or *stationary* result and that this theorem holds for positive rewards such as profits and for negative rewards such as costs. Let us now ascertain the long run fraction of time that our CPFL is in either the fallow or in the non-fallow state.

2.2 The fallow and the non-fallow states

Consider a dynamic and probabilistic CPFL that is used for swidden agriculture and that can be represented by the stochastic process $\{X(t) : t \geq 0\}$. At any given point in time, this CPFL (stochastic process) exists either in state 1 or the fallow state or in state 2 or the non-fallow state. If the CPFL is currently in state i, $i = 1, 2$, then—as a result of human and natural factors described in Sect. 1—it moves to the next state after an exponentially distributed amount of time with mean $1/\lambda_i$, $\lambda_i > 0$, for $i = 1, 2$. The next state is state 1 (the fallow state) with probability p_1 and state 2 (the non-fallow state) with probability $p_2 = 1 - p_1$.

Our task now is to use the renewal-reward theorem (equation (1)) to determine the long run fraction of time that our CPFL is in state i for $i = 1, 2$. To this end, note that the CPFL or the stochastic process $\{X(t) : t \geq 0\}$ is *regenerative*. In particular, we can say that a cycle commences each time our CPFL makes a state transition. Further, the expected length of this cycle is finite and it equals

$$E[\text{length of cycle}] = \frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}. \tag{2}$$

Moving on, for a particular state i, suppose that our CPFL earns a reward of one when it is in this state and a reward of zero when it is in the other state. Then, some thought tells us that the expected reward per cycle is given by a particular ratio and that ratio is

$$E[\text{reward per cycle}] = \frac{p_i}{\lambda_i}, \quad i = 1, 2. \tag{3}$$

Now, applying the renewal-reward theorem (equation (1)) to the problem at hand, the long run fraction of time our CPFL is in state i, $i = 1, 2$, is given by

$$\frac{p_i/\lambda_i}{(p_1/\lambda_1) + (p_2/\lambda_2)} \tag{4}$$

with probability one.[2]

[2]Note that the results contained in (3) and (4) do not depend on our assumption regarding rewards in the paragraph preceding (3). Given that we are working with a *regenerative* process—see the paragraph

Differentiating (4) with respect to $\lambda_i > 0$, $i = 1, 2$, we see that as λ_i increases, the mean time spent by our CPFL in state i or $1/\lambda_i$ decreases and hence the long run fraction of time spent by our CPFL in state i also *decreases*. In addition, the reader should note that for our CPFL, the length of a cycle has a continuous distribution. Therefore, the stationary *probability* of being in state i for $i = 1, 2$ or $\lim_{t\to\infty} \text{Prob}\{X(t) = i\}$ exists and is, in fact, equal to (4) or the long run fraction of *time* that the CPFL is in state i. This completes our determination of the long run fraction of time that the CPFL under study is in either the fallow state 1 or in the non-fallow state 2. We now proceed to compute the stationary probability that the excess variable associated with the stochastic process $\{X(t) : t \geq 0\}$ representing our CPFL exceeds a particular non-negative value.

2.3 The stationary probability

To sharpen our discussion thus far, suppose that the stochastic process $\{X(t) : t \geq 0\}$ describing our CPFL in Sect. 2.2 is actually a *renewal* process. We now have to model the interoccurrence times for this renewal process. To this end, note first that the interoccurrence times are clearly *non-negative* random variables. Second, the interoccurrence times will obviously be affected by the mean time spent in the fallow and in the non-fallow states and, in general, we expect these two mean times to be *unequal*. Putting these two pieces of information together and keeping our Sect. 2.2 analysis in mind, it makes sense to describe the interoccurrence times with a *mixture* of two exponentials with *different* means. As noted by Tijms (2003, pp. 446–447), the distribution of such a mixture is known as a hyperexponential distribution of order 2 and it is typically denoted by H_2. In our case, we suppose that the density of the pertinent H_2 distribution is given by $p_1\lambda_1 e^{-\lambda_1 t} + p_2\lambda_2 e^{-\lambda_2 t}$.[3]

Suppose now that we observe our CPFL at some arbitrary time t. Without loss of generality, assume that our CPFL at time t is in the fallow state 1. Then, probabilistically speaking and from a practical perspective, we would be very interested in knowing how much longer our CPFL will stay in the fallow state before it makes a transition to the non-fallow state. To answer this question it will be necessary to compute what is known in the renewal theory literature as the "excess variable".[4] Further, because we are interested in shedding light on the sustainability of swidden agriculture in a probabilistic environment, we shall take a long run view of our computation

preceding (2)—our method for determining the long run fraction of time that this regenerative process spends in a particular state is standard. See Ross (2003, pp. 425–426) for a textbook discussion of this point.

[3]The $\{X(t) : t \geq 0\}$ stochastic process from Sect. 2.2 is now a renewal process. For this renewal process, a renewal occurs each time the process makes a state transition. In addition, a cycle starts each time the process makes a state transition. This means that a cycle does *not* consist of the time spent in the fallow *and* in the non-fallow states. Finally, because we are working with a renewal process, the sequence of interoccurrence times are statistically independent and identically distributed. This last point is true for all distributions including the hyperexponential distribution with which we are working in this section.

[4]Formally, the excess variable is defined to be the time elapsed from epoch t until the next renewal after epoch t. See Ross (1996, pp. 116–117) or Tijms (2003, pp. 37–38) for textbook discussions of the excess variable.

of this excess variable. Mathematically, what this means is that we shall be computing the *stationary probability* that the excess variable associated with the stochastic process $\{X(t) : t \geq 0\}$ representing our CPFL exceeds a given non-negative value.

Let us denote the excess variable of interest by $Y(t)$ and the given non-negative value referred to in the previous paragraph by $x \geq 0$. Then, in symbols, we want to compute $\lim_{\{t \to \infty\}} \text{Prob}\{Y(t) > x\}$. Recall our description of the hyperexponential distribution with density $\sum_{\{i=1\}}^{2} p_i \lambda_i e^{\lambda_i t}$ in the first paragraph of this section. From this we can tell that our CPFL (renewal process) can be represented by the $\{X(t) : t \geq 0\}$ process from Sect. 2.2 with the understanding that a renewal occurs each time the $\{X(t) : t \geq 0\}$ process makes a state transition. Now, by conditioning and then using the lack of memory of the exponential distribution,[5] we reason that

$$\text{Prob}\{Y(t) > x\} = e^{-\lambda_1 x}\text{Prob}\{X(t) = 1\} + e^{-\lambda_2 x}\text{Prob}\{X(t) = 2\}. \tag{5}$$

From the discussion in the last paragraph of Sect. 2.2, we know that $\lim_{\{t \to \infty\}} \text{Prob}\{X(t) = i\}$ for $i = 1, 2$ exists and is given by (4). Using this result, we can take the limit as time approaches infinity and simplify the right-hand-side (RHS) of (5). This gives us an expression for the stationary probability we seek and this expression is

$$\lim_{\{t \to \infty\}} \text{Prob}\{Y(t) > x\} = e^{-\lambda_1 x}\left[\frac{p_1\lambda_2}{p_1\lambda_2 + p_2\lambda_1}\right] + e^{-\lambda_2 x}\left[\frac{p_2\lambda_1}{p_1\lambda_2 + p_2\lambda_1}\right], \quad x \geq 0. \tag{6}$$

Equation (6) tells us that the long run probability that the excess variable associated with our CPFL will be greater than a particular non-negative value $x \geq 0$ is given by the weighted sum of two exponential terms. The weights themselves are given by (4) and they equal the long run probability that our CPFL is in each of the two possible states. This completes our discussion of the two questions that comprise the subject matter of this paper.

3 Conclusions

In this paper, we used renewal theory to shed light on two hitherto unstudied questions concerning the fallow and the non-fallow states in swidden agriculture. Specifically, we first used the theory of renewal-reward processes to determine the long run fraction of time that a CPFL is in either the fallow or in the non-fallow state. Second, we used the hyperexponential distribution to compute the stationary probability that the excess variable associated with the stochastic process representing our CPFL exceeds a given non-negative value.

The analysis in this paper can be extended in a number of directions. Here are two suggestions for extending the research delineated in this paper. First, it would

[5] Let X be an exponentially distributed random variable that represents the lifetime of a certain item. Then, if the residual life of this item has the same exponential distribution as the original lifetime, regardless of how long this item has already been in use, then X has the memoryless property. See Ross (1996, pp. 37–38) or Tijms (2003, pp. 440–441) for textbook expositions of the memoryless property of the exponential distribution.

be useful to generalize the analysis presented here by focusing on a model in which there are $n > 2$ states and one of these many states is the fallow state. With such a model, one could examine, for instance, a profit maximization exercise for a small farmer in which the time spent by the CPFL in the fallow state is at least as large as some exogenously given value. Second, it would also be useful to eschew the use of the exponential distribution and study the extent to which one can get analytical results with a model in which the interoccurrence times discussed in Sect. 2.3 have an arbitrary distribution. Studies of swidden agriculture in a stochastic setting that incorporate these features of the problem into the analysis will provide additional insights into a particular agricultural practice that has significant economic and ecological implications.

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